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The stringy lattice models

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Abstract. We propose a stringy lattice model as a natural generalization of the usual lattice model, which is parallel to a generalization from point particle theory to string theory in particle physics. We examine several examples, one of which describes a non-ideal gas of strings, which changes to string liquid at some boiling temperature. As a most important example we study the light cone string field theory. We make a lattice version of it and study its properties as a statistical model. It can be regarded as a very high-dimensional statistical model in which mean-field approximation is powerful. The long string sector is shown to be dynamically trivial and the low curvature strings are statistically favoured.

1. Introduction

Lattice models are now of great interest to researchers in various fields of physics and mathematics. Fields concerned are the study of computer experiments in gauge theory and some statistical systems and the study of constructive field theory and exactly solvable lattice models. Studies of the latter topic are now of special interest since many unexpected connections to other fields such as conformal field theory, theory of knots and links, Chern-Simons theory, theory of quantum groups, theory of elliptic functions, etc have been discovered so far.

In this paper we try to generalize the lattice models naturally in the following way. Let us prepare a square lattice first. In the usual lattice models we place a dynamical variable on each site or link of the lattice, for the Ising case the variable σ_x for each site x. Because the variables are placed locally, we call these ordinary models 'point-like lattice models'.

This time we consider the set of all the loops drawn in the square lattice. For each loop X, we place the dynamical variable σ_X. As usual in order to define the Hamiltonian based on the concept of nearest neighbours (NN) the definition of NN in the set of loops is needed. Taking some definition, we set the Hamiltonian as

$$H = -J \sum_{\langle X, Y\rangle} \sigma_X \sigma_Y \tag{1.1}$$

where X and Y are loops, $\langle X, Y\rangle$ indicates that X is a NN of Y. σ_X is a dynamical variable taking the values ± 1 and J is a coupling between two loops. Given the Hamiltonian as above the partition function is defined as

$$Z(\beta) = \sum_{\text{config}} \exp[-\beta H(\text{config})] \tag{1.2}$$

where the configuration is specified if we assign $+1$ or -1 for every loop X.

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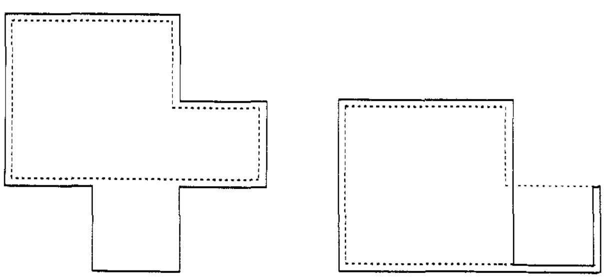
Since a point x of σ_x in the usual Ising model has now been generalized to a loop X of σ_X, we call this kind of model the 'stringy lattice model'. This kind of extension of the point-like model is possible in most cases. We study several examples of this kind and apply it to the study of string field theory in this paper. In the next section we give a precise definition of a stringy Ising model, and study it in detail. It will be shown that some truncated version of it is reduced to the usual Ising model, so exactly solvable in 2D. Furthermore using the lattice gas picture, this system is shown to describe a non-ideal gas of strings so it changes to a string liquid at some boiling temperature. Some interesting features are found in the loop-loop correlators in this system. In section 3, we will investigate the lattice light cone string field theory as a typical example of the stringy lattice models. Although the problem of the continuum limit is not accessed, we will reveal several important properties inherent in this model. First of all this theory is shown to be regarded as a very high-dimensional point-like lattice model in which mean-field approximation is powerful. One interesting consequence of this model is the fact that strings longer than some critical length are dynamically trivial. Furthermore it is pointed out that low curvature strings have a much larger value of the critical length. Possible application of the mean-field application is discussed therein. The reader interested in only the application to string field theory can pass on to section 3. In section 4, we will present a somewhat exotic model of the stringy lattice model. This is actually defined in 2D and shown to be equivalent to the six-vertex model. In section 5 we conclude this paper.

2. Stringy Ising model

2.1. Introduction

First of all let us prepare a D-dimensional ($D \geq 2$) square lattice Γ whose size is finite. Let $\mathscr{L}_p^{\mathrm{Is}}$ denote the set of all the loops on Γ. Degenerate loops which go through the same link more than once are also included in $\mathscr{L}_p^{\mathrm{Is}}$. Given a loop X in $\mathscr{L}_p^{\mathrm{Is}}$ we can deform one of the links of X as shown in figure 1. We consider this as a fundamental deformation†, which always increases the length of the loops by two. Note that a fundamental deformation like figure 1(b) is not an exception of this rule. Based on this definition of the fundamental deformation we define the NN (nearest neighbour) of this model as in figure 1.



(a) (b)

Figure 1. Examples of the fundamental deformation.

† This type of deformation was previously used in the study of gauge theory in [1].

Two loops X and Y are NN if and only if one of these is the fundamental deformation of the other. With this definition of NN, the Hamiltonian and partition function of the stringy Ising model are defined as (1.1) and (1.2) respectively. Since degeneration of loops is allowed, loops concerned in (1.1) are of any length, even though we restrict Γ to be finite. This makes the system a little difficult to treat. To catch the feeling of what is going on, we temporarily restrict the length of the loops.

We consider the model with its loops restricted to $L = 2, 4, \ldots, L_0$; we call it the L_0 model in the remainder of this section. As a first tractable example let us discuss the $L_0 = 4$ model. Loops involved are those in the first two lines of figure 2. It is easily seen that only 2_1 and 4_1 interact. Loops 4_2 to 4_4 have no effect in the system since none of them are the fundamental deformation of 2_1; they do not appear in the Hamiltonian and we can completely ignore them.

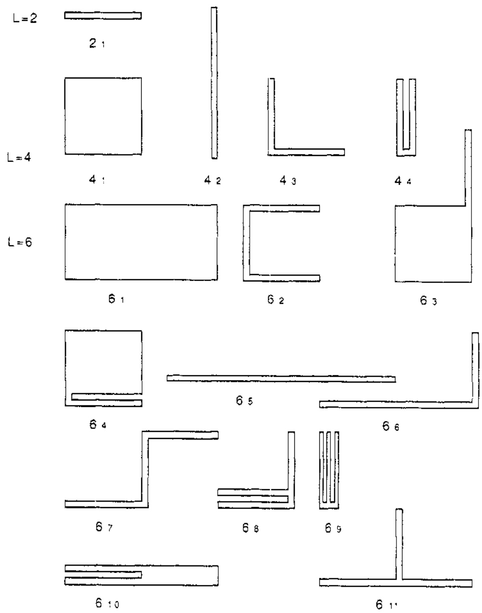


Figure 2. An exhaustive list of loops with length $L = 2$, 4, 6, ignoring the place and the direction in which they are placed.

Loop 4_1 is placed at each face on the square lattice Γ and loop 2_1 at each link. Every 4_1 has four 2_1's as its NN and each 2_1 is shared by neighbouring 4_1's. Thence the coupling of the system in 2D is pictured as figure 3(a) where a circle and a square represent loops 2_1 and 4_1 respectively and each line represents the existence of the coupling between the loops. We call this kind of diagram a coupling lattice in the following.

We will now show, using the well known techniques in lattice models [2], that the $L_0 = 4$ model is essentially reduced to the Ising model, therefore exactly solvable in $D = 2$.

Let 2_1's be indexed by i and two neighbouring 4_1's of i, by i' and i''. The partition function of $L_0 = 4$ model reads

$$Z^{(4)} = \sum_{\text{config}} \prod_i \exp[K\sigma_i(\sigma_{i'} + \sigma_{i''})]. \tag{2.1}$$

where $K = J\beta$.

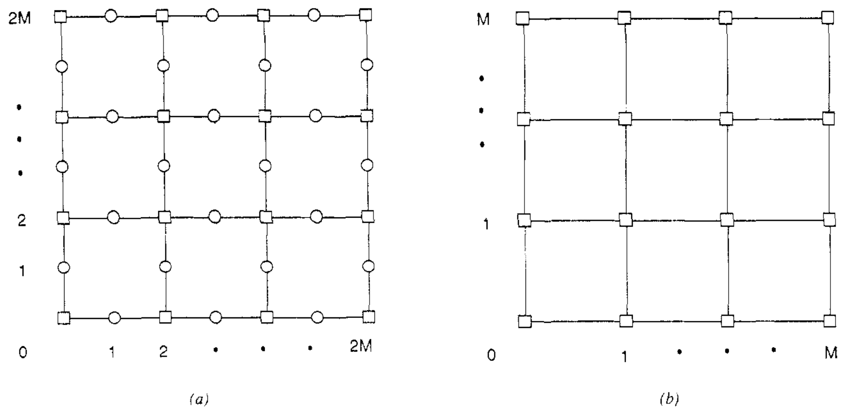


Figure 3. (a) A coupling lattice for $L_0 = 4$ model. (b) A lattice made from (a) by removing circles in it.

Summing over $\sigma_i = \pm 1$ first in the configuration sum, $\exp[K\sigma(\sigma_{i'} + \sigma_{i''})]$ changes to $2\cosh[K(\sigma_{i'} + \sigma_{i''})]$ which is rewritten as $R\exp[A\sigma_{i'}\sigma_{i''}]$ if we choose A and R as $R = 2e^A$, $e^{2A} = \cosh 2K$. Consequently $Z^{(4)}$ is rewritten as

$$Z^{(4)} = 2^{2M(M+1)}(\cosh 2K)^{M(M+1)} \sum_{\text{config}} \exp\left[A \sum_{\langle i,j\rangle} \sigma_i\sigma_j\right] \tag{2.2}$$

where $2M$ is a one-dimensional size of the lattice Γ in figure 3(a), and the configuration sum is taken over spin variables placed on only the squares in figure 3(a). So the lattice is now changed to figure 3(b). If we denote a free energy per unit site by $\psi(K)/\beta$, it is derived through a little calculation that

$$\begin{aligned}\psi^{(4)}(K) &= \lim_{\text{size}\to\infty} \frac{-1}{\#\text{site}} \ln Z^{(4)} \\ &= \psi^{\text{Ising}}(S) + \frac{-1}{3}\ln(4\cosh(2K)).\end{aligned} \tag{2.3}$$

Using the equation determining T_c of the Ising model $\sinh(2A_c) = 1$ [2] (subscript c denoting critical values), we now can calculate the T_c of the $L_0 = 4$ model, as $\cosh(2K_c) = 1 + \sqrt{2}$, this leads to

$$T_c = \frac{J}{0.764\ldots} \tag{2.4}$$

which is about a half of that in the Ising model, $T_c = J/0.440\ldots$. This result is naively anticipated on sight of the coupling lattice in figure 3(a). Since the coupling is loose in this model compared to the Ising model, we need to lower the temperature to form a long-range order so that T_c is lower now.

Now we go on to the $L_0 = 6$ model in which the length of loops are restricted from two to six. In this case the loops 6_5 to 6_{11} in figure 2 are completely irrelevant as were 4_2 to 4_4 in the $L_0 = 4$ case. The new feature now is that the system separates to two mutually non-coupling sectors, one of which consists of 2_1, 4_1, 6_1 and 6_2, the other consists of 4_2, 4_3, 4_4, 6_3 and 6_4.

Consider the first sector; there arises another new feature that 6_2 is also irrelevant in the calculation of the partition function although it appears in the Hamiltonian. We postpone the presentation of the reason for this for a while, it will be given in a more general framework later. To make the coupling transparent, we form the coupling lattice first to get figure 4(*a*). This first sector of the $L_0 = 6$ model also essentially reduces to the Ising model through the same discussion as for the $L_0 = 4$ model, so the dimensionless free energy is written as

$$\psi^{(6),\mathrm{I}}(K) = \psi^{\mathrm{Ising}}(2K^*) - \tfrac{2}{5}\ln(16\cosh(2K)) \tag{2.5}$$

where $e^{2K^*} = \cosh 2K$, and superscript I indicates the first sector. T_c is calculated as $\cosh^2(2K_c) = 1 + \sqrt{2}$ (see the equation just above (2.4)), leading to $T_c = J/0.495\ldots$. This value is between that of the Ising model and that of the $L_0 = 4$ model.

The second sector of the $L_0 = 6$ model is a little complex. The coupling lattice of it is shown in figure 4(*b*), where an open circle, a triangle, an open square, a solid square and a solid circle denote 4_2, 4_3, 6_3, 6_4 and 4_4, respectively.

We cannot solve it exactly, but we can naively expect that the critical temperature of the system is higher than that in the Ising model because the coupling lattice of it is tighter now.

Those systems discussed so far in fact have fairly good physical interpretation provided we use the lattice gas picture [2, 3] known in the usual Ising model. Let us review it shortly.

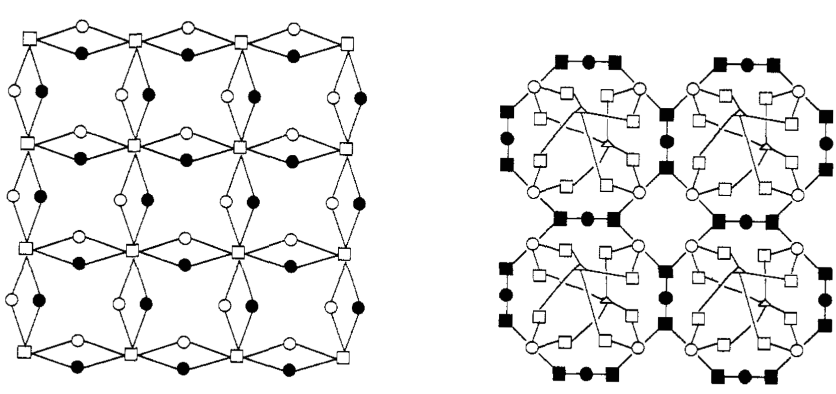
Consider a gas composed of molecules interacting through the Lennard-Jones potential

$$\phi(r) = 4\varepsilon\left[\left(\frac{r_0}{r}\right)^{12} - \left(\frac{r_0}{r}\right)^{6}\right] \tag{2.6}$$

where ε, r_0 are constants. Idealizing this by a square-well potential

$$\phi(r) = \begin{cases} +\infty & \text{for } 0 \leqslant r \leqslant r_0 \\ -\varepsilon & \text{for } r_0 \leqslant r \leqslant r_1 \\ 0 & \text{otherwise} \end{cases} \tag{2.7}$$

and replacing a continuum space by a fine lattice does not affect the qualitative feature of the system. Let s_i denote the number of molecules in a lattice site i. Then the grand



(*a*) (*b*)

Figure 4. (*a*) A coupling lattice for the first sector of $L_0 = 6$ model. A square, an open circle and a solid circle represent the loops 4_1, 2_1 and 6_1, respectively. (*b*) A coupling lattice for the second sector of $L_0 = 6$ model.

canonical partition function of the system reads

$$Z = \sum_s \exp\left[\beta(n\mu - \sum_{\langle i,j\rangle} \phi_{ij} s_i s_j)\right] \tag{2.8}$$

where the interaction energy is $\phi_{ij} = 1$ only if i is a NN of j and zero otherwise. $n = s_1 + \ldots + s_N$ is the total number of particles with N being the total number of sites. μ is a chemical potential. We have ignored the contribution from the kinetic part which is separated individually so it only has smooth dependence on β. s_i assumes 0 or 1 due to the presence of the hard core of the square well potential.

If we use the relation $\sigma_i = 2s_i - 1$, Z can be rewritten as

$$Z = C^N \sum_{\text{config}} \exp\left[\beta J \sum_{\langle i,j\rangle} \sigma_i \sigma_j + \beta H \sum_i \sigma_i\right] \tag{2.9}$$

with some constant C. Correspondingly this provides us with

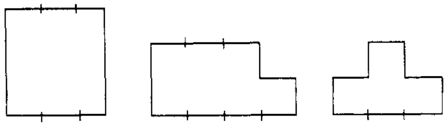
$$\begin{aligned} \rho &= \tfrac{1}{2}(1 + M(H)) \\ P &= -DJ + H - \frac{1}{\beta}\psi \end{aligned} \tag{2.10}$$

etc, where ρ, P are the density and pressure of the fluid, respectively, and D denotes the dimensionality of the space where the gas is placed. H is an external magnetic field and M is magnetization.

We can easily recognize, from the first expression displayed above, that the discontinuous change of magnetization $M(H)$ from $-M_0$ to M_0 at T_b lower than T_c corresponds to the discontinuous decrease of the density at T_b with pressure fixed in the gas picture. So this surely describes the phase change from gas to liquid (first order with respect to H). T_b is nothing but a boiling temperature. This story generalizes to the stringy case without any obstacles.

Now we consider the string gas; strings are attracting each other if two loops are just about close enough to degenerate each other, but they cannot degenerate exactly due to the hard core of the square well potential. (Make sure not to confuse these strings with those appearing later in the framework of string field theory.) For instance the $L_0 = 4$ model describes the situation that the $L = 2$ and $L = 4$ loops are interacting through that potential, and at T_b the string gas condensates to the string liquid. In the case of the $L_0 = 6$ model, which is composed of two decoupled fluids, the boiling temperatures are different from each other. If we lower the temperature the second sector condensates to liquid first, then comes the first sector.

Next consider the general case L_0 model. This time we start a sufficiently low temperature in which all the loops are in the liquid phase. The fluid may be composed of several mutually non-interacting components, each of which has a different boiling temperature. Which component of loops begins to boil first? The answer is the sector containing regular, long loops. The reason follows now. The longest loops can only interact with the loops shorter by two (not longer by two), so they have fewer partners to couple in general, especially if they are regular, where regular means that they have fewer convexes and concaves. This situation is most conveniently exhibited by a picture shown in figure 5 which represents the example of the $L_0 = 12$ case. All the loops in figures (a), (b), (c) are the longest loops but figure (b) has only one NN and figure (a) has none, therefore the coupling lattice is quite loose leading to low T_b. As the convexes and concaves increase as in figure (c), the number of partners of it increases and the coupling lattice becomes tighter to raise the boiling temperature.



(a) (b) (c)

Figure 5. Examples of the l-loop (longest loop) in the $L_0 = 12$ model. (a) is more regular than (b), and (b) is more regular than (c).

So, the physical picture of the general L_0 model is that as we increase the temperature the string liquid experiences boiling at many T_b's before everything boils up.

Let us consider here the behaviour of the L_0 model in the limit $L_0 \to \infty$. In this situation it is expected that the boiling temperatures of all the sectors become higher, and diverge as L_0 tends to infinity since the coupling lattice becomes tighter.

On the other hand it is generally expected that mean-field approximation becomes exact for the sufficiently-high-dimensional lattice models. In fact, in the square lattice Ising model, it is known that critical exponents take the value calculated by the mean-field approximation for any $D \geqslant 4$ [4]. (It has been proved [5] that ($D \geqslant 2$)-dimensional Ising models experience second-order phase transition with respect to β at finite T_c, irrespective to the value of D.)

An important lesson presented by the study of point-like lattice models is a concept of effective dimension [2]. In the regular D-dimensional lattice, the number of sites reached by n steps from some fixed site is proportional to n^D. Conversely for a given lattice, if that number $\sim n^D$, we say that the effective dimension of the lattice is D. The effective dimension accounts for how many sites, around one site, influence the site. It is naively recognized that if the effective dimension is large, mean-field approximation becomes exact and the system goes into the classical region in which critical exponents take the values determined by the mean-field approximation, i.e. $\nu = 0$, $\beta = \frac{1}{2}$, $\gamma = 1$, $\eta = 0$, etc.

If L_0 is large enough, each sector of the L_0 model is regarded as a high-dimensional Ising model as is clear from its coupling lattice. So it is a natural expectation that the L_0 model has second-order phase transition, belongs to the universality class of classical region and T_c is high but finite if L_0 is.

2.2. *Loop–loop correlation*

Now we see interesting new features found in the loop–loop correlators. Consider the L_0 model. We refer to the longest loops ($L = L_0$) as l-loops and the others as s-loops. We remark that since the number of loops expands exponentially in length L, l-loops are the majority in number.

Now we try to classify l-loops.

Suppose we calculate the partition function

$$Z = \sum_{\text{config}} \exp\left(\sum_{\langle X, Y \rangle} K\sigma_X\sigma_Y \right). \tag{2.11}$$

Concentrate on the contribution from the l-loops, namely terms $K\sigma_X\sigma_Y$ with Y: l-loop and X: s-loop. If Y is not the fundamental deformation of any s-loops X, then σ_Y does not appear in the Hamiltonian. This kind of l-loop is called an ignored loop. For

$K\sigma_X\sigma_Y$ to be non-zero, the difference between X and Y must be some face α as shown in figure 6(a). We denote this situation by $Y-X=\alpha$. Then develop the high-temperature expansion [2] using $\exp(K\sigma_X\sigma_Y)=\cosh K+\sigma_X\sigma_Y\sinh K$,

$$\begin{aligned}\exp\left[K\sum_{Y:\, l\text{-loop}}\sigma_X\sigma_Y\right]&=\prod_\alpha\exp\left[K\sum_{X-Y=\alpha}\sigma_X\sigma_Y\right]\\&=(\cosh K)^n\prod_\alpha\prod_{X-Y=\alpha}(1+u\sigma_X\sigma_Y)\\&=(\cosh K)^n\prod_\alpha\left(1+u\sum_{X_0-Y_0=\alpha}\sigma_{X_0}\sigma_{Y_0}+u^2\sum_{X_i-Y_i=\alpha}\sigma_{X_0}\sigma_{Y_0}\sigma_{X_1}\sigma_{Y_1}+\ldots\right)\end{aligned}\tag{2.12}$$

where $u=\tanh K$ and n is the number of pairs (X, Y) such that $X-Y=\alpha$. When there is only one X which satisfies $X-Y=\alpha$ for a fixed Y, it is at most once that σ_Y appears in each product $\sigma_{X_0}\ldots\sigma_{Y_i}$ in the summand. Thus summing over $\sigma_Y=\pm1$ erases the terms which contain σ_Y so we can ignore such loops from the begining in the calculation.

The l-loop Y which contributes to the calculation must satisfy the following property. There exist s-loops X, X' and faces α, α' such that

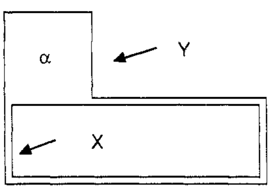
$$X-Y=\alpha \qquad X'-Y=\alpha' \qquad X\neq X'. \tag{2.13}$$

Under these conditions α may be equal to α'. Examples of the $\alpha\neq\alpha'$ case and $\alpha=\alpha'$ case are exhibited in figure 6(b) and figure 7 respectively. Those loops satisfying (2.13) are called dependent loops. The l-loops which are neither ignored nor dependent are called independent loops.

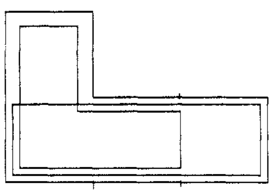
Now the partition function is reduced to

$$Z=C'^N\sum_{\text{config}}\exp[K\,\Sigma'\,\sigma_X\sigma_Y] \tag{2.14}$$

with C' being some constant and N being the number of independent loops. Σ' is over all the loops whose length are less than or equal to L_0 and the l-loops involved are

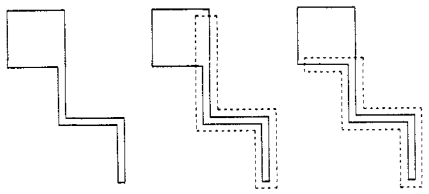


(a)



(b)

Figure 6. (a) A picture representing $X-Y=\alpha$. (b) One example of a dependent loop in the case $\alpha\neq\alpha'$.



(a) (b) (c)

Figure 7. An example of a dependent loop in the case $\alpha = \alpha'$.

dependent. Let us consider a specific example. Take the $L_0 = 4$ model, which is indeed quite simple. One $L = 4$ loop is of the type in figure 7 (dependent) and the others all ignored. Then comes the $L_0 = 6$ model. 6_1 is clearly a loop of the type in figure 6(b), and now we can understand why we could ignore 6_2, which was actually an independent loop. This concludes the first sector.

Next in the second sector two l-loops 6_3, 6_4 are of the type in figure 7 and the others are ignored loops. We can make the general remark that even though an independent loop 6_2 can be ignored in the calculation Z, it cannot be forgotten in the calculation of correlators. We use a notation $\sigma^{6_1}_{i,x}$ for the spin variable on the loop 6_1 placed at x with i being an index specifying the direction in which the loop is placed.

We show in the following that all the correlators are calculated from the correlators involving short and dependent loops only. First we try to calculate the correlator of $\sigma^{6_1}_{i,x}\sigma^{6_2}_{j,y}$, using the technique of high temperature expansion. Note that 6_2 is an independent loop. Now the term containing $\sigma^{6_2}_{j,y}$ which vanished in the calculation of Z no longer vanishes due to the presence of one more $\sigma^{6_2}_{j,y}$ in front of the Boltzmann weight. This results in

$$\langle \sigma^{6_1}_{i,x}\sigma^{6_2}_{i,y} \rangle = u \langle \sigma^{6_1}_{i,x}\sigma^{4_1}_{y} \rangle \tag{2.15}$$

where $u = \tanh \beta J$.

For higher correlators we only have to make a replacement

$$\sigma^{6_2}_{j,y} \to \sigma^{4_1}_{y} \tanh \beta J. \tag{2.16}$$

Because an independent l-loop has an unique s-loop as its NN, we call it a conjugate s-loop. Then the general rule is stated that all the correlators are obtainable by the correlators involving only s-loops and dependent loops. To calculate the correlator involving independent loops we only have to make the replacement

$$(\text{independent } l\text{-loop}) \to (\text{conjugate } s\text{-loop}) \tanh \beta J. \tag{2.17}$$

This result clarifies the behaviour of the independent loops. In high temperature $\beta \to 0$, independent loops (such as 6_2 in the $L_0 = 6$ model) are completely unseen since all the correlators containing it vanish, but in the lower temperature $\beta \to 1$, they behave exactly as their conjugate s-loop. Note that this interesting property is valid for any L_0 models.

Next we investigate more closely the $L_0 = 4$ model and the first sector of the $L_0 = 6$ model in two dimensions in order to check how their reductions to the Ising model are complete. Since the two models are essentially the same we only discuss the former.

We have previously reduced the calculation of Z of this model to the Ising model. We now try to calculate its correlators. Since the variables $\sigma^{4_1}_{x}$ are independent of the reduction, correlates between 4_1's are written as

$$\langle \sigma_x^{4_1} \sigma_{y}^{4_1} \rangle = \langle \eta_x \eta_y \rangle_{\text{Ising}} \tag{2.18}$$

where η_x is a usual Ising spin variable placed on sites of the lattice in figure 3(b). In the case where the site 2_1 is involved it is derived, using the same technique leading to (2.15), that

$$\langle \sigma_x^{4_1} \sigma_{y}^{2_1} \rangle = \langle \eta_x (\eta_{y'} + \eta_{y''}) \rangle_{\text{Ising}} \tanh 2A \tag{2.19}$$

where $A = \frac{1}{2}\ln \cosh 2\beta J$, y', and y'' are two NN (squares) of y (a circle), see figure 3(a). For arbitrary correlators the calculation rule is

$$\begin{aligned} &\sigma_x^{4_1} \to \eta_x \\ &\sigma_y^{2_1} \to (\eta_{y'} + \eta_{y''}) \tanh 2A \end{aligned} \tag{2.20}$$

namely: correlation with 2_1 can be replaced by correlation with the sum of the two neighbouring 4_1s, multiplied by tanh $2A$.

We have now found that all the correlators are calculated in the framework of the usual Ising model, but it contains an interesting new operator $\sigma_x^{2_1}$ in addition to the usual spin.

3. Light cone string field theory

In this section we discuss the lattice light cone string field theory as a typical example of the stringy lattice model. In order to clarify the discussion we restrict ourselves to the closed string theory. Generalization to the open string theory is a straightforward task.

Naive discretization of the theory is given below. However, it is a non-trivial and difficult problem as to whether the continuum limit can be safely taken for this lattice string theory. Accordingly we restrict ourselves here to the study of the lattice string only as a statistical system. If the continuum limit can safely be achieved without drastically disturbing properties of the lattice string exposed below, results of the following analysis apply to the continuum string field theory.

Euclidean action of the closed light cone string field theory is given as [6]

$$\begin{aligned} S = \int & \mathrm{d}X^+ \, \mathrm{d}p^+ [\mathrm{d}X^j] \\ & \times \left\{ \alpha' p^+ \Phi^* \frac{\partial}{\partial X^+} \Phi + p^+ \int_0^{2\pi p^+} \mathrm{d}\sigma \left[\alpha' \left| \frac{\delta \Phi}{\delta X^j(\sigma)} \right|^2 + \frac{1}{\pi^2 \alpha'} \left(\frac{\partial X^j(\sigma)}{\partial \sigma} \right)^2 |\Phi|^2 \right] \right\} \\ & + g \int \mathrm{d}X^+ \, \mathrm{d}p^+ \, \mathrm{d}q^+ \, \mathrm{d}r^+ [\mathrm{d}X^j][\mathrm{d}Y^j][\mathrm{d}Z^j] \delta_{(XYZ)} \\ & \times \Phi^*(X^j, p^+, X^+) \Phi(Y^j, q^+, X^+) \Phi(Z^j, r^+, X^+) + \text{cc} \end{aligned} \tag{3.1}$$

where

$$\Phi = \Phi(X^i, p^+, X^+)$$

and

$$\Phi^* = \Phi^*(X^i, p^+, X^+)$$

are string fields; $X^1(\sigma), \ldots, X^{D-2}(\sigma)$ are transverse modes of a string, X^+ is a light cone time and p^+ is a longitudinal momentum of a string, $\delta_{(XYZ)}$ ensures the right connection and the momentum conservation of the three strings, α' is a inverse string tension.

Using the same kind of procedure as in the previous sections, the $D_0 = (D-2)$-dimensional loops can be naturally discretized as follows. We first change the continuous D_0 space (for transverse mode) to a square lattice whose spacing is equal to a. Recall that $\int [\mathrm{d}X]$ means the sum over all the loops in $\boldsymbol{R}^{D_0}$. On the discretization of D_0-dimensional space we discretize the loops in the space so that they only run through the links of the lattice (see figure 2).

Now all the discretized loops must be summed up in order to give a lattice action. Let us note that degenerate loops in which the same link is run through more than once are not excluded, since those loops naturally appear throughout the discretization of a functional differentiation discussed later.

Now the discrete loop $X(\sigma)$ made of $L = L(X)$ links (which we call the L sector) are parametrized with $\sigma_i = (I/L)i$ $(i = 0, 1, \ldots, L-1)$ with $I = 2\pi p^+$. Therefore the loops in the L sector have $2D_0L$ directions for the fundamental deformation because each link can be deformed in $2D_0$ directions. This implies that X has just $2D_0L$ nearest neighbours. This means that in the vicinity of the loop X with $L(X) = L$, the dimension of the loop space is given as D_0L.

Therefore $[\mathrm{d}X]$ in the L sector is represented as a^{D_0L} upon the discretization. So we rescale the field as

$$\Phi(X) = a^{-D_0L(X)/2}\phi_X \tag{3.2}$$

to cancel this volume factor in the lattice action. If Φ did not have this kind of dependence on L, large L sectors would quickly collapse or diverge due to the volume factor $a^{D_0L(X)}$, which reduces the system to a trivial one. It is clearly true that all the bilinear terms of Φ are saved by that rescaling, but it is amazing that the trilinear interaction term is also the case. The functional integration joining the three strings erases $\delta_{(XYZ)}$, $[\mathrm{d}Y]$ and $[\mathrm{d}Z]$. The remaining $[\mathrm{d}X]$ gives $a^{D_0L(X)}$ and trilinear Φ fields give $a^{-D_0L(X)/2}a^{-D_0L(Y)/2}a^{-D_0L(Z)/2}$. Recalling the string connection condition $L(X) = L(Y) + L(Z)$, the powers of a sum up to zero. This means that the Φ^3 interaction also survives the rescaling mentioned above without collapse or divergence. This property is kept even if we take the vertices, including open strings which are trilinear or quadrulinear in Φ, into account. The reason is that the sum of lengths of loops is always conserved.

The action contains a functional derivative, and the differentiation with respect to a loop implies an infinitesimal deformation of the loop changing their length while, as is seen from (3.2), the factor $a^{-D_0L(X)/2}$ is essentially responsible for a reaction of Φ under an infinitesimal change of the length of X. Thus in order to take care of the derivative term we actually carry out the rescaling of the field before the discretization. Regarding a to be a free parameter first, insert (3.2) into the second term in (3.1), which leads to

$$\int [\mathrm{d}X]\, \mathrm{e}^{2K} \oint \mathrm{d}\sigma \left[\left(\frac{\delta K}{\delta X^j(\sigma)} \right)^2 |\phi_X|^2 + \frac{\delta K}{\delta X^j(\sigma)} \frac{\delta |\phi_X|^2}{\delta X^j(\sigma)} + \left| \frac{\delta \phi_X}{\delta X^j(\sigma)} \right|^2 \right] \tag{3.3}$$

where $K = (-D_0 \ln a)\hat{L}(X)/(2a)$ with $\hat{L}(X) = aL(X)$ being the true length of a loop X. On the discretization, a is identified as a lattice spacing at which e^{2K} is cancelled with $[\mathrm{d}X]$.

To begin with we see the outline of the discretization of the last term of (3.3). We recall that the discretization of a Laplacian term for the usual D-dimensional point particle field theory, i.e. $\int \mathrm{d}^D x\, \partial^i\phi(x)\partial^i\phi^*(x)$, gives

$$\sum_x 2D|\phi(x)|^2 - \sum_{\langle x,y\rangle} (\phi(x)\phi^*(y)+\mathrm{cc}) \tag{3.4}$$

with x and y running through the lattice points. The second sum is over all the NN in $\boldsymbol{Z}^D$. Similarly discretization of the last term of (3.3) which is a generalization of Laplacian is considered to give

$$\sum_X 2D_0L(X)|\phi_X|^2 - \sum_{\langle X,Y\rangle} (\phi_X\phi_Y^* + \mathrm{cc}) \tag{3.5}$$

for each L sector, where $\langle X, Y\rangle$ denotes the NN in the sense of loops. We remark that the NN here now include the types as in figure 8, so as to take into account the deformation along a loop. These are not discarded since we have already solved the constraint erasing longitudinal oscillation.

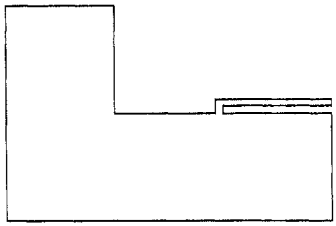


Figure 8. Fundamental deformations along the loop itself.

Now we consider (3.5) more carefully. The functional differentiation we now consider is represented by

$$\frac{\delta\phi_X}{\delta X^j(\sigma_i)} = \frac{\mathrm{d}}{\mathrm{d}t}\,\phi_{X(\sigma)+te_j\delta(\sigma-\sigma_i)}\bigg|_{t=0} \tag{3.6}$$

with $\boldsymbol{e}_j$ being a unit vector along the j axes. To carry out the discretization of the right-hand side of the above equation, we first change the delta function to a regularized one, $h_i(\sigma)$, which is defined as $h_i(\sigma) = L/I$ if $\sigma_i \leqslant \sigma < \sigma_{i+1}(\sigma_{i+1}-\sigma_i = I/L)$, otherwise zero. Then the discretization of (3.6) is given as

$$(\phi_{X(\sigma)+te_jh_i(\sigma)} - \phi_{X(\sigma)})/t = (\phi_{X_{ij}} - \phi_X)/(Ia/L). \tag{3.7}$$

We now have t equal to $t = Ia/L$ (small) so as to let $X(\sigma) + te_jh_i(\sigma)$ be equal to X_{ij} which is defined to be a loop obtained by shifting a link i, $i+1$ in the direction of jth axis by just one lattice unit. X_{ij} ($i = 0, 1, \ldots, L$; $j = 1, 2, \ldots, D_0$) form NNs of X having length $L(X)+2$.

Next we calculate $\delta\hat{L}(X)/\delta X(\sigma)$ which is in the first and second term in (3.3). This functional differentiation can be carried out explicitly recalling $\hat{L}(X) = \oint \sqrt{X'^2(\sigma)}\,\mathrm{d}\sigma$ to give

$$\frac{\delta\hat{L}(X)}{\delta X^j(\sigma)} = \frac{(\boldsymbol{X}'\cdot\boldsymbol{X}'')X^{j\prime}}{|\boldsymbol{X}'|^3} - \frac{X^{j\prime\prime}}{|\boldsymbol{X}'|}. \tag{3.8}$$

This expression, evaluated at $\sigma = \sigma_i$, is equal to $(4L(X)/Ia)\omega_{Xij}$ where $\omega_{Xij} = 1(-1)$ only if $\sigma = \sigma_i$ is a corner of the discrete loop X, and the vector $\boldsymbol{X}(\sigma_i) - \boldsymbol{X}(\sigma_{i-1})$ is (anti-)parallel to the jth axis respectively, otherwise zero. This implies that $\delta\hat{L}(X)/\delta X(\sigma)$ gets contributions only where the loop has concentrated curvature (in other words at a corner). This can easily be understood if we notice that $\delta\hat{L}(X)/\delta X(\sigma) = 0$ is an equation for a geodestic curve.

After these calculations, the last term in (3.3) turns into

$$\sum_{i=0}^{L-1} \frac{I}{L} \sum_{j=1}^{D_0} |(\phi_{X_{ij}} - \phi_X)/(Ia/L)|^2 \tag{3.9}$$

where the sum of i and I/L is derived from $\int d\sigma$. Writing down the final result for the lattice version of full theory requires further notation. There is a possibility that two or more of X_{ij}'s for different (ij) accidentally meet since we include the degenerate loop. Thus we define the quantity deg(X, Y) as the number of X_{ij} such that $X_{ij} = Y$ for fixed X and Y, moreover we define $\eta(X)$ as a sum of deg(Y, X) for all NN. Y of X shorter than X by two. Let note that, for the generic loop, deg(X, Y) is of the order of 1; however for the most degenerate loop deg(X, Y) is proportional to $L(X)$.

The first term of (3.3) creates the quantity

$$\overline{\omega_X^2} = \frac{1}{L(X)} \sum_{i,j} \omega_{X_{ij}}^2 = \frac{1}{L(X)} (\text{the number of corners of } X). \tag{3.10}$$

A generic loop, in which the number of corners is proportional to L, has $\overline{\omega_X^2} \sim 1$, on the other hand a low curvature loop as $\overline{\omega_X^2} \sim 0$. Furthermore we set $\omega(X, Y) = \omega_{ij}$ for $Y = X_{ij}$, and $\omega(X)$ is defined to be a sum of deg$(Y, X) \times \omega(Y, X)$ for all NN loops Y shorter than X by two. This concludes the discussion on the discretization of the second term of (3.1).

Lastly we shall also change $x^{\pm} = x^0 \pm x^{D-1}$ directions to lattices of spacing a and size M leading to the discretization of the momentum $p^+ = (2\pi/Ma)k$, $k = 0, 1, 2, \ldots$. We express $\phi_{X, p^+ = (2\pi/Ma)k, X^+ = la}$ as $\phi_{X,k,l}$ in the following, but suppress unimportant arguments for simplicity.

The third term in (3.1), $(\partial X^j(\sigma)/\partial\sigma)^2$, is easily discretized to $(a/(I/L))^2$, and the first and the last of (3.1) are also easily discretized.

We finally get the following expression of a lattice action $\hat{S}$.

$$\begin{aligned}
\hat{S} = \hat{S}_0 + \hat{S}_{\text{int}} = \frac{2\pi}{M} \sum_{k=0}^{\infty} \sum_{l \in Z} \Bigg(& \frac{\alpha' \pi k}{Ma^2} \sum_X \phi_{X,l}^* (\phi_{X,l+1} - \phi_{X,l-1}) \\
& + \frac{\alpha'}{8\pi a^4} \sum_X \Bigg\{ (4D_0 \ln a)^2 \overline{\omega_X^2} - 4D_0 \ln a \frac{(L(X)-2)\omega(X)}{L(X)^2} \\
& + \frac{(L(X)-2)\eta(X)}{L(X)^2} + 2D_0 + \frac{4a^4}{\pi^2 \alpha'^2} \Bigg\} \hat{L}(X)^2 |\phi_X|^2 \\
& + \frac{-\alpha'}{8\pi a^4} \sum_{\langle X, Y \rangle} \deg(X, Y) a \hat{L}(X) \phi_X^* \phi_Y + \text{cc} \\
& + g \sum_{\langle X, Y, Z \rangle} \frac{2\pi}{Ma} \sum_{k'=0}^{k} \phi_{X,k}^* \phi_{Y,k'} \phi_{Z,k-k'} + \text{cc} \Bigg).
\end{aligned} \tag{3.11}$$

The most important feature of this statistical model is that this is a very high-dimensional statistical model with non-nearest-neighbour interaction. In the high-dimensional lattice model the mean-field approximation is quite a powerful method. For example, the universality class of a high-dimensional lattice model is a trivial one determined by the mean-field approximation, which is known from the renormalization group analysis [4]. One possible discussion, applying the mean-field approximation, is presented at the end of this section.

We now see that the long string sector is dynamically trivial and the low curvature sector is statistically favoured. Suppose that we take a configuration sum (path integral) with sources

$$Z_0(J, J^*) = \int \prod_{Xkl} \mathrm{d}\phi_{Xkl}\, \mathrm{d}\phi^*_{Xkl} \exp\left[-\hat{S}_0 + \sum_{Xkl} (\phi_{Xkl} J^*_{Xkl} + \phi^*_{Xkl} J_{Xkl}\right] \tag{3.12}$$

with only the free part of the action $\hat{S}_0$. The exponential factor determines the Gaussian distribution of the field configurations of ϕ_{Xkl}'s. In sight of $\hat{S}_0$ in (3.11), we find that as X gets longer, the coefficients of the square terms in $\hat{S}_0$ increase as $\hat{L}^2$ while those of the cross terms at most linear in $\hat{L}$. Thence for sufficiently large $\hat{L}(X)$, standard deviations and average values of ϕ_X distribution are vanishing, i.e. ϕ_X is sharply concentrated at zero. This allows us to trivially integrate out these degrees of freedom. Let us achieve this below.

First, sectors with different k's are not coupled in $\hat{S}_0$ so that Z_0 can be written as $Z_0 = \Pi_k Z_{0,k}$. Here we discuss the generic case in which $\omega_X^2 \sim 1$, $\eta(X) \sim L$ and $\omega(X) \sim L$. If we assume the lattice spacing a is small, the dominant coefficient of $|\phi_X|^2$ is the first one, $(\alpha'/8\pi a^4)(4D_0 \ln a)^2 \omega_X^2 \hat{L}^2$. The critical length $\hat{L}_c$ above which the integration is trivial is determined by equating this and $\alpha'\pi k/Ma^2$ or $(\alpha'/8\pi a^3)\hat{L}$ assuming deg ~ 1. The greater value of $\hat{L}_c$ is given by the former. This $\hat{L}_c$ is proportional to $\sqrt{k}$. For later convenience we make a safer choice. We actually integrate ϕ_X on loops longer than $\hat{L}_c(k)$ defined as

$$\hat{L}_c(k) = \begin{cases} A_0 & \text{for } 0 \leqslant k \leqslant k_0 \\ -A + Bk & \text{for } k_0 < k \end{cases} \tag{3.13}$$

with suitable positive constants A_0, A, B and k_0. We call the loops longer than $\hat{L}_c(k)$ long loops, and call the others short loops.

After the integration with respect to ϕ_{Xkl} for long X, we get

$$Z_0(J, J^*) = C \exp\left[\sum_{\text{long}} f_{Xl;Yl',k} J_{Xkl} J^*_{Ykl'} \right] Z_{0,\text{short}}(J, J^*). \tag{3.14}$$

C is a divergent quantity independent of J, and the last factor contains short loops only. Coefficients f are nothing but the propagators between the two loops. The values quickly decay as $f \sim \exp[-\gamma n]$ where n symbolically represents the difference between two loops. For the time direction, $n = |l - l'|$ and $\gamma \sim \ln\{[M(D_0 \ln a)^2/ka^2]\hat{L}(X)^2\}$. For the transverse space direction, n is the number of fundamental deformations to reach Y from X and $\gamma \sim \ln[(D_0 \ln a)^2/a]\hat{L}(X)]$. So the longer loops described by ϕ_{Xkl} will be quickly decoupled because $\hat{L}(X)$ gets greater.

So far we have only been concerned with the free part. Let us switch on the Φ^3 interaction. Using the well known formula together with the form of $Z_0(J, J^*)$ presented above, the total partition function is calculated to be

$$\begin{aligned} Z(J, J^*) &= \exp[-\hat{S}_{\text{int}}(\partial/\partial J, \partial/\partial J^*)] Z_0(J, J^*) \\ &= C \exp\left[\sum_{\text{long}} f_{Xl;Yl',k} J_{Xkl} J^*_{Ykl'} \right] Z_{\text{short}}(J, J^*) \end{aligned} \tag{3.15}$$

with $\hat{S}_{\text{int}}(\phi, \phi^*)$ being the Φ^3 interaction term. $Z_{\text{short}}(J, J^*)$ is the full partition function (including Φ^3 interaction) containing ϕ and J associated to short loops only. The second equality is proved noting the following fact. When a string X separates to Y and Z, and X is a long string, namely $L_c(k_X) < L(X) = L(Y) + L(Z)$, either Y or Z must be long to avoid the contradiction of $L(Y) < L_c(k_Y)$ plus $L(Z) < L_c(k_Z)$. Therefore at least two of the differential operators in $(\mathrm{d}/\mathrm{d}J_X^*)(\mathrm{d}/\mathrm{d}J_Y)(\mathrm{d}/\mathrm{d}J_X)$ is the one associated to a long string. Thence after the commutation of this operator with the second factor of the second line in (3.14), there survives at least one differential operator associated to a long one which vanishes when acting on Z_{short}. This tells us that the long-loop sector suffers no effects from Φ^3 interaction. Thus we have arrived at the result that the long strings are dynamically trivial in the full theory. This fact is consistent with the naive expectation. Since the action (3.1) describes the string with constant line density $1/\alpha'$, long strings are highly massive. Thence the above observation is thought to be the decoupling theorem of higher massive modes.

Let us mention less generic cases. If the curvature of the string is small enough $\overline{\omega_X^2}$ and $\omega(X)$ almost vanish. Thus the dominant term in the coefficient of a square term changes to the third or the forth one in the square coefficient. This makes $\hat{L}_c$ much longer. Thus we can say that low curvature strings will survive with far longer lengths than generic one. This is physically natural because low curvature strings, thought to have less oscillation energy, then correspond to a less massive mode than a generic one.

So far we have argued the case of strings with no drastic degeneracy. For highly degenerate strings in which the degeneracies $\deg(X, Y)$ are proportional to the length of the string one of the coefficients of the cross term $\deg(X, Y)\hat{L}$ becomes quadratic in $\hat{L}$. Thus the average value does not approach to zero as $\hat{L}$ gets greater, although the standard deviation does. It seems to imply the condensation of ϕ_X to some value, but this natural physical interpretation is not yet reached.

Before closing this section we present here our aim to explore non-pertubative effects in string theory. The action we would like to treat is of the form

$$\hat{S} = \sum_{X,kl} A(a)|\phi_X|^2 + \sum_{\langle XY\rangle,k,l} B(a)\phi_X\phi_Y^* + \sum_{X,k,l} C(a)\phi_l\phi_{l+1}^*$$
$$+ \sum_{\langle XYZ\rangle,k,k',l} G(a)\phi_{X,k}\phi_{Y,k'}^*\phi_{Z,k-k'}^*. \tag{3.16}$$

In this expression $A(a)$, $B(a)$, $C(a)$ and $G(a)$ are, in reality, determined by the condition that all the correlation functions remain finite under $a \to 0$. And actually, A, B, C and G may depend on the other information such as $L(X)$, k, etc, but we neglect it here for simplicity. As we noted before, the most significant feature inherent in this model is its high dimensionality. Let us pay attention to the second term in (3.16) first, for fixed X whose length is L, and which contains the sum over all the NN Y of X. There are many NNs with length $L+2$ like a cloud around X, so we replace all ϕ_Y^* by the same value $f^*(L+2)$ for all Y with $L(Y) = L+2$. Similarly we replace ϕ_{l+1}^* by $f^*(L)$, and $\phi_{Y,k'}^*\phi_{Z,k-k'}^*$ by $f^*(L')f^*(L-L')$. Then we are left with

$$\hat{S} = \sum_{X,k,l} [A(a)|\phi_X|^2 + D_0 LB(a)\phi_X f^*(L+2) + C(a)\phi_X f^*(L)$$
$$+ \sum_{k',L'} G(a)\phi_X f^*(L')f^*(L-L') + \text{cc}]. \tag{3.17}$$

Now the integral $\int \Pi\, \mathrm{d}\phi_{Xkl}\, \mathrm{d}\phi_{Xkl}^* \exp[-\hat{S}]$ is easily achieved, since it is simply a product of non-coupled Gaussian integrals. By this procedure free energy $F = (-1/\beta) \ln Z$ is determined as a functional of $f(L)$, $F = F[f]$. So the minimum condition determines the functional form $f_0(L)$ which represents the distribution of length of this loop system. Moreover if we coupled the external metric $G^{\mu\nu}$ in the kinetic term of the original action, after some suitable procedure of the discretization, we would be able to get the free energy as the functional like $F = F[f, G^{\mu\nu}]$. This minimum condition is, in principle, considered to determine the metric which is actually realized in the world of string theory.

These last comments on the application of mean-field theory are still speculative at present and a detailed investigation is now under way.

4. A different example of the stringy lattice models

Finally we present a completely different example of the stringy lattice models in 2D. This model does not have any constraints on lengths of its loops in a thermodynamic limit. Further, this model will be shown to be equivalent to the six-vertex model [2, 7].

We prepare a 2D finite square lattice Γ. We define a loop space $\mathscr{L}_p^{6V}$, this time, as a set of all the loops drawn in the square lattice Γ such that no loops degenerate. Let us see what the configuration is. For any loop $X \in \mathscr{L}_p^{6V}$ we can assign 1 (excited) or 0 (non-excited). Some constraints on its configuration are imposed as in the case of several point-like lattice models such as vertex models, IRF models, etc. The first constraint is that two excited loops cannot share the same links and the second one is that any intersecting points of excited loops including self-intersection must be one of the types shown in figure 9.

Then the energy for a given configuration is defined as

$$H = \sum_{\langle X, Y\rangle} F_i(X, Y) + \sum_X F_s(X) \tag{4.1}$$

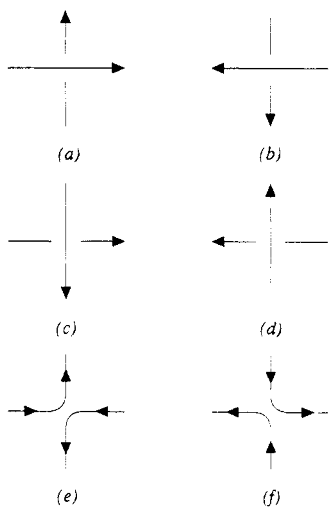


Figure 9. Allowable types of intersection of loops.

where

$$F_{\mathrm{i}}(X, Y) = \begin{cases} n_A A + n_B B + n_C C & \text{if } X \text{ and } Y \text{ are excited} \\ 0 & \text{otherwise} \end{cases}$$

$$F_{\mathrm{s}}(X) = \begin{cases} n_A A + n_B B + n_C C & \text{if } X \text{ is excited} \\ 0 & \text{otherwise} \end{cases}$$

with n_A, n_B, n_C being the number of intersections between X and Y or X itself of the types in figure 9(a, b), (c, d), (e, f) respectively. This concludes the definition of the model.

Then we find that two arrows go into a site and two arrows come out of a site for each site (vertex). This is nothing but a configuration of the six-vertex model [7]. So the configurations of our model correspond to those in the six-vertex model in a one-to-one manner. Also energy assignment coincides if we choose a, b, c as

$$a = \sinh(\gamma - \theta) = \mathrm{e}^{-\beta A} \qquad b = \sinh \theta = \mathrm{e}^{-\beta B} \qquad c = \sinh \gamma = \mathrm{e}^{-\beta C}. \tag{4.2}$$

It is well known that the above Boltzmann weight is a solution to the Yang–Baxter equation so that the model is exactly solvable.

We take the case $C < A$, B. Then the doubly-degenerate ground-state configurations are as shown in figure 10 and that shifted by one unit. Some lower excited configurations are shown in figure 11. As the temperature goes up, long excited loops increase and at some T_{c} the system goes into random phase.

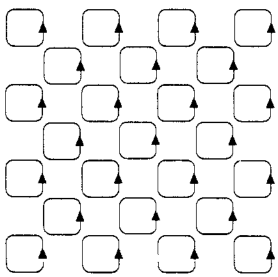


Figure 10. A ground-state configuration of the model in the case $C < A$, B.

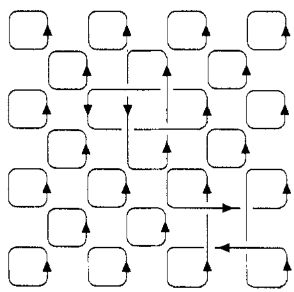


Figure 11. An example of a lower excited configuration.

Let us make one remark before concluding the section. These types of stringy lattice models are found elsewhere. The concept of construction of the above model is actually based on the low-temperature expansion of the model, which was, for example, known for the Ising model. So we can also regard the Ising model as a kind of stringy lattice model in that way. The point is that we cannot regard these types of models as a dynamics of strings as in the case of our first model in place of excitation of loops, because there is no term corresponding to a chemical potential.

5. Conclusion

We have proposed stringy lattice models in this paper. As a first example, the stringy Ising model was defined and the simplest truncated version was solved in 2D. This model was fairly well interpreted physically as a string fluid system. Interesting behaviour of the longest loops (l-loop) was found through loop–loop correlators. In section 3 we examined the lattice light-cone closed-string field theory as a typical model of the stringy lattice models. The problem of the continuum limit was not clarified, but only the properties as a lattice model were studied. That system was pointed out to be a very high-dimensional point-like lattice model. The analysis has shown that the long-string sector was dynamically trivial and that low curvature strings are statistically favoured, which were physically natural. One speculation of applying high dimensionality to the model was given. In section 4 a somewhat exotic example was displayed. It was constructed based on the concept of high- or low-temperature expansion of the usual point-like lattice models. A special feature of that kind of model was that there was no constraint on the length of the loops.

This paper only provides us with an initial setting and a little examination in the approach to stringy lattice models. There is much to be done in this direction; for instance, making the other examples of the stringy lattice models and to investigate them. We think we will be able to expose a rich and interesting world through this kind of approach as was so in the point-like lattice models. This is because the generalization is such a natural one. Also we could carry out a study of $\mathscr{L}_p$. We know mathematically interesting structures were found in the usual continuum loop space [8]. A parallel discussion seems possible for the discrete version.

Of all the problems we think the most important one is the application to string field theory. This model is regarded as a statistical system with very high effective dimension. So this system is in the ideal situation for the application of the mean-field approximation. We expect that mean-field approximation will give us a promising strategy for the study of the non-perturbative nature of string theory.

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